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Simulation of Bacterial Growth Process from Animal Waste in a
Discontinuous Bioreactor

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Abstract

The global emphasis on environmental conservation and the urgency to shift towards sustainable energy sources has brought significant focus on renewable energy alternatives. Consequently, there's been a pronounced interest in exploring unconventional solutions to replace traditional fuels. One such innovative approach involves the development of discontinuous bioreactors designed specifically to harness methane gas from animal waste. In response to the pressing need for eco-friendly energy solutions, this research has delved into modelling and simulating a discontinuous bioreactor system. This system aims to efficiently convert animal waste into methane gas—a pivotal step towards sustainable energy generation. The study explores the potential of this bioreactor technology as a feasible and environmentally conscious means to produce renewable energy while addressing the challenges posed by conventional fuel usage. The study relied on Monod kinetics, a mathematical model, to establish the correlation between the growth rate of microorganisms and the concentration of substrate available. This model offered a detailed insight into how the growth of microorganisms responds to varying substrate concentrations. Moreover, to simulate the complex dynamics of substrate consumption and the concurrent production of microorganisms and methane gas, an advanced numerical method known as the fourth order Rong-Kuta method was employed. This numerical approach allowed for a comprehensive understanding of the intricate interactions within the system, shedding light on the interplay between substrate utilization, microbial growth, and methane gas generation. By using this method, the study could effectively simulate and analyse the multifaceted processes occurring within the bioreactor system, offering valuable insights into its functioning and potential optimizations. The study explored the impact of the initial microorganism concentration on methane production. Starting with initial substrate and microorganism concentrations of 74.51 g/L and 61.1 g/L, respectively, the investigation revealed a deviation of approximately 53.8 percent between the mathematical model and the actual laboratory data. As per the model, the anticipated methane production after 70 days is estimated to be 29.10 g/L. The rate of substrate decomposition and methane gas production is contingent upon the substrate's residence time. Elevating the initial microorganism concentration accelerates methane gas production within a shorter duration. However, the total amount of methane produced remains unaffected by the initial concentration of microorganisms.

The model showcased in this study holds the capacity to forecast the duration needed for the reaction, optimize the performance of bioreactors, aid in the design of essential process equipment, and facilitate the scaling up of equipment—such as storage tanks. This comprehensive tool not only enables the prediction of optimal reaction times but also offers insights into refining the process for producing high-purity methane in larger volumes within bioreactor systems. It's instrumental in ensuring the efficient operation of bioreactors while accommodating the necessary controls and adjustments to enhance methane production at scale without compromising quality.

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INTRODUCTION

In today's world, getting energy to remote places, especially in countries like Iran, is a big challenge. It's not easy and costs a lot to extend traditional energy sources to these areas. That's why more and more people are looking into alternative ways to produce energy, and one method that's gaining a lot of attention is using biomass. Biomass is made from things like leftover crops, organic waste, and special energy plants. It's a great solution because it's not only environmentally friendly, reducing our dependence on finite fossil fuels, but it also helps with the environmental problems caused by burning those fuels.

The cool thing about biomass is that it's versatile and can be sourced locally, which means we don't have to worry about transporting traditional fuels to remote areas. There are advanced technologies that can turn biomass into energy, using methods like anaerobic digestion, gasification, and combustion. This shift towards biomass energy production is really promising, especially for places with limited access to energy. It's not just about meeting immediate energy needs but also about reducing greenhouse gas emissions and taking care of the environment.

Biochemical processes play a big role in turning biomass into usable energy. One of the methods, anaerobic fermentation, produces something called biogas, which is mostly made up of methane—a valuable fuel. This process is a sustainable way to create energy, taking organic waste and turning it into something useful.

Looking at the composition of biogas, the table shows that more than half of it is methane. This makes biogas a significant and promising source of fuel, giving us a cleaner and more renewable option compared to traditional fuels. The high methane content in biogas makes it a real contender in the search for greener and more sustainable energy solutions. It's not just about meeting our energy needs; it's about doing it in a way that's better for the planet and our wallets. So, biogas is playing a crucial role in moving towards a more environmentally conscious and economically feasible energy future.

Gas Name	Percentage composition	Formula
Methane	55 to 75	CH ₄
Carbon dioxide	35 to 45	CO ₂
Nitrogen	0 to 1	N ₂
Hydrogen	0 to 1	H ₂
Hydrogen sulphide	1 to 2	H ₂ S
Oxygen	Insignificant amounts	O ₂
Carbon monoxide	Insignificant amounts	CO

Table 1: Biogas Composition

NOMENCLATURE

0.1 Nomenclature

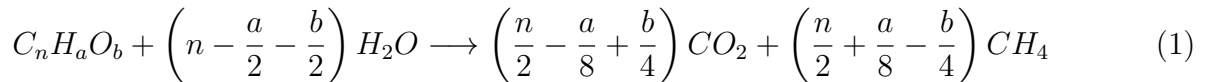
Symbols	Meaning
μ	Microorganisms produced per day
K_d	Mortality rate in day^{-1}
K_s	Semi-saturation constant
μ_{max}	Maximum growth rate for microorganisms
μ_{net}	Net growth rate of microorganisms per day
X	Concentration of microorganisms
S	Substrate concentration
X_0	Initial concentration of microorganisms
S_0	Initial substrate concentration
P	Concentration of methane gas
Y_p	Efficiency of Methane production
Y_{xis}	Microorganisms' production efficiency

PROBLEM FORMULATION

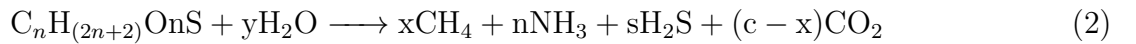
0.2 Origin

Biological methane generation, also known as anaerobic fermentation, happens spontaneously in places like swamps, sewers, and landfills. Important feedstocks for the production of biogas include sewage, animal dung, industrial, and municipal wastes, as well as agricultural residues like sugar beet and rice bran. There is a great deal of potential for producing biogas because these resources are widely available in our nation. Currently, biogas is used to produce energy and heat on a smaller scale in several highly industrialized nations around Europe as well as in nations such as Canada. Beyond just producing electricity, biogas also has the benefit of being cost-effective, producing very little sludge, extracting minerals, reducing odours from waste products, and reducing greenhouse gas emissions. 1952 saw the proposal of a methodology by Baswell and Müller which suggested that with a maximum error margin of 5 percent, the amount of methane and carbon dioxide produced could be calculated using the composition of the organic matter fed into the reactor. By estimating the reactor's gas output, this method offered a practical way to forecast biogas production with a respectable degree of accuracy depending on the composition of the organic input.

In 1976, Boyle modified equation and proposed the following reaction, in which he determined the amount of ammonia and hydrogen sulphide production by considering nitrogen and sulphur.



Equation (1) represents the chemical reaction for biogas production.



where the coefficients are defined as follows:

$$x = \frac{1}{8}(4c + h - 2o - 3n - 2s) \quad (3)$$

$$y = \frac{1}{4}(4c + h - 2o - 3n - 3s) \quad (4)$$

In 1998, Baserga categorized organic matter into carbohydrates, fats, and proteins, calculating gas production and methane fractions separately for each. Kimmer and Schilcher built upon Baserga's model, refining it to account for substrate-specific decomposition rates. Similarly, Amun

et al. used organic matter as a benchmark to gauge methane energy content from materials like cereals and corn, tailoring their model for plant-based substrates. Kimmer and Schilcher’s model focuses on animal substrates, while Amun et al.’s is geared towards plant materials. The pressing need to replace fossil fuels has spurred extensive research to bolster biogas production and enhance methane purity. Studies demonstrate that pH control profoundly impacts methane production, elevating efficiency by 7.6 times compared to anaerobic digestion pH levels. Organic acids generated during digestion not only hinder methane production but also induce severe environmental acidification. Moreover, exploring the effect of nanoparticles on biogas production revealed promising findings. Utilizing nickel nanoparticles amplifies methane production by 2.17 times compared to nanoparticle-free conditions. These investigations underscore innovative approaches like pH manipulation and nanoparticle integration as pivotal strategies to optimize biogas production, signalling a significant step towards sustainable energy solutions.

There’s a notable scarcity of research in modeling and simulating biogas production reactors, but several pioneering studies have made significant strides in various substrates and reactor types. In 2011, Zhou et al. presented a robust model tailored for agricultural waste substrates, demonstrating its reliability in stable laboratory fermentations. Their model not only aligned well with laboratory data but also proved predictive for agricultural biogas production units. Lee et al., also in 2011, focused on modeling a biogas reactor catering to fats, oils, greases, and kitchen waste. Their results highlighted how cosobestra significantly expedite the organic bio-degradation process, particularly reducing delay stages. They showcased that utilizing kitchen waste, fats, and oils notably augmented methane gas production within their model. Grader et al. explored methane production efficiency trends during corn fermentation, establishing a clear relationship between methane output and the chemical composition and quality of forage used. Beba and Atley, through modeling a discontinuous biogas production reactor using agricultural waste, identified the contusion model as the best fit, effectively correlating growth rate and substrate decomposition. Furthermore, investigations expanded to semi-continuous biogas production from municipal waste, leveraging artificial intelligence for biogas production from shrimp lake sediments, and exploring both discontinuous and continuous reactors for biogas production from olive pomace. These studies collectively highlight the diverse scope and depth of research in modeling biogas production across various substrates and reactor configurations, showcasing the multifaceted approaches and applications within this burgeoning field.

0.3 Assumptions

The assumptions in the provided text for modeling a continuous biogas production reactor from animal waste are:

Constant Volume Operation: It is often assumed that this occurs under constant volumes; therefore, modeling approaches based on volume variations are eliminated.

Constant Temperature and pH: Assuming the temperature and pH of the reaction media is 35 degree celcius and 7, respectively, Thus, in the modeling it is assumed that it remains constant over time.

Conversion of Volatile Solids: All volatile solids will be transformed into biogas. The validity of this assumption allows a clear link between the feedstock and biogas generation.

Proportional Loss of Volatile Solids: It is thought that the decrease in volatility is proportional to biogas creation. This, as assumed, is due to the integrated processes involved in the degradation of solubilized substrates and gas production.

Simultaneous Substrate Consumption and Biogas Production: Accordingly, this text shows that substrate consumption and biogas generation happen together in a biogas reactor.

Such an oversimplifying assumption is, however, employed in order to reduce the complexity of modelling operations.

Dependency on Residence Time: The rate of substrate decomposition and biogas production are assumed, for purposes of this study, to be functions of the residence time of the input substrates. This points out the role of time as an input factor which plays a significant part towards the effectiveness of the process.

The presented simplifications represent an approximate way of viewing the interactions between organisms, substrates, and gas generation during the biological process of manure decomposition.

0.4 Required Equations

The general mass balance for microorganisms in a discontinuous reactor is as follows:

$$\frac{d[X]}{dt} = \mu_{net}[X] \quad (5)$$

$$\mu_{net} = \mu - K_d \quad (6)$$

$$\mu = \frac{\mu_{max}[S]}{K_s + [S]} \quad (7)$$

In equation (5) $[X]$ is the concentration of microorganisms, μ_{net} of microorganisms' net growth rate, and t is time. The net growth rate of microorganisms shown in equation (6) is defined as the difference between the microorganisms produced and the microorganisms that got killed.

In equation (6) K_d is the microbial mortality rate and μ is the microorganism growth rate from equations (5) and (6). German scientists Michael and Menten provided the foundation for predicting the kinetics of bacterial growth. According to their 1913 model, the concentration of the substrate affects enzyme activity. Similar to the Michael-Menten model, Monod demonstrated a nonlinear relationship between the concentration of the substrate and the rate of microbial growth. We can see the Monod model in equation (7).

where K_s is the semi-saturation constant, $[K]$ is the substrate concentration, and μ_{max} is the maximum growth rate of microorganisms. Microorganisms' particular growth rate is determined by the concentration of their substrate as well as other environmental factors including pH, inhibitors, and the warmth. After the completion of the Monod model in the next years, more Whole models worked well in contexts with a variety of substrates. For both simple substrates and pure culture mediums, the Monod model exhibits remarkable accuracy. This approach works well in homogeneous contexts; it is not appropriate for complicated substrates or inhomogeneous situations. Since the creation of biogas from animal waste is discussed in this study Animal manure is thought to be a simple substrate and has been examined, therefore the Monod model has been applied to the models. Equation (8) represents the changes in the concentration of microorganisms in the discontinuous reactor after equation (5) is simplified and the use of equations (6) and (7) within it.

$$\frac{d[X]}{dt} = \left(\frac{\mu_{max}[S]}{K_s + [S]} - K_d \right) [X] \quad (8)$$

$$\frac{d[S]}{dt} = -\frac{\mu[X]}{Y_{xis}} \quad (9)$$

The general mass balance of the substrate for discontinuous processes is in equation (9) Equation (10) represents the production efficiency of microorganisms, which is determined by dividing the number of bacteria generated by the amount of substrate consumed.

$$Y_{vis} = \frac{[X] - [X_0]}{[S_0] - [S]} \quad (10)$$

$$(11)$$

Assuming that methane gas is created, the subtitle "o" refers to the starting concentration. When the substrate is consumed concurrently; the formula for variations in the concentration of. The following describes the methane gas generated in the discontinuous reactor.

$$\frac{d[P]}{dt} = Y_P \mu [X] \quad (12)$$

where Y_p is the production efficiency of methane gas and $[P]$ is its concentration. Equations (9), (8), and (12) are solved concurrently in order to calculate the amount of methane produced in the reactor. Using the Runge-kutta method of fourth order approach , these equations can be solved. The reference laboratory's results are compared with the simulation results .

NUMERICAL ANALYSIS

0.5 Runge-Kutta method

The equations are as follows:

$$Y_{n+1} = Y_n + k, \quad (13)$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad (14)$$

$$k_1 = h \cdot f(x_n, Y_n), \quad (15)$$

$$k_2 = h \cdot f\left(x_n + \frac{h}{2}, Y_n + \frac{k_1}{2}\right), \quad (16)$$

$$k_3 = h \cdot f\left(x_n + \frac{h}{2}, Y_n + \frac{k_2}{2}\right), \quad (17)$$

$$k_4 = h \cdot f(x_n + h, Y_n + k_3). \quad (18)$$

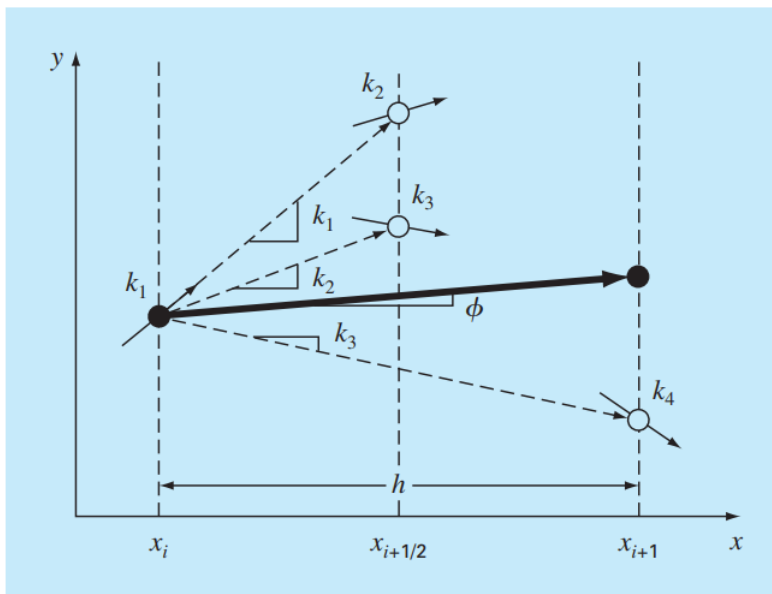


Figure 1: Graphical depiction of the slope estimates comprising the fourth-order RK method

0.6 Euler Method

$$y_{i+1} = y_i + hf(x_i, y_i), \quad (19)$$

$$y_{i+1} = y_i + hf(x_{i+1}, y_{i+1}), \quad (20)$$

$$y_{i+1} = y_i + \frac{h}{2} [f(x_i, y_i) + f(x_{i+1}, y_{i+1})]. \quad (21)$$

Euler Explicit Method equation (19)

Euler Implicit Method equation (20)

Semi Implicit (Crank Nicholson) Method (21)

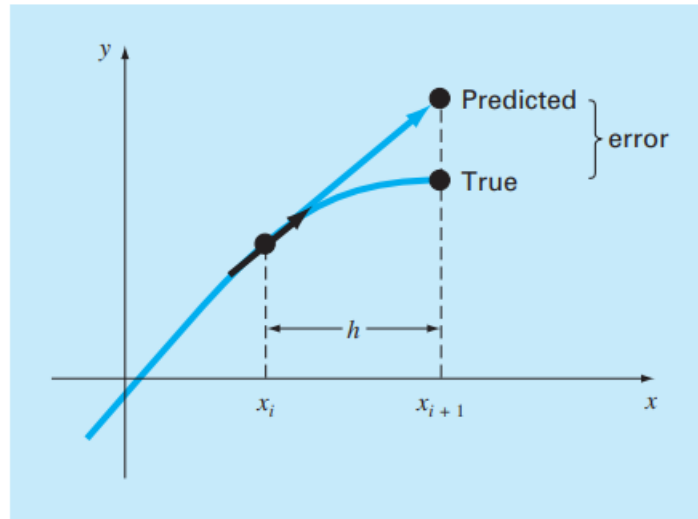


Figure 2: Euler method

0.7 Huen's Method

$$y_{i+1} = y_i + f(x_i, y_i)h, \quad (22)$$

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y_{0i+1})}{2}h. \quad (23)$$

Predictor equation (22)

Corrector equation (23)

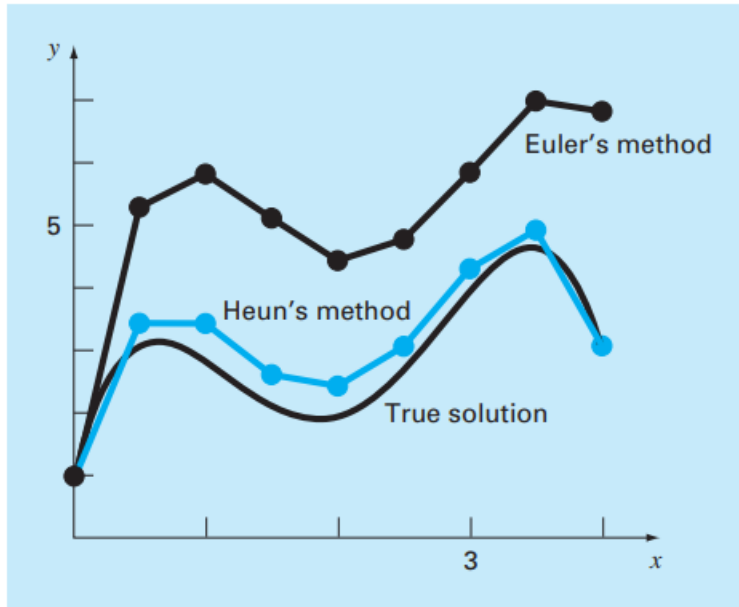


Figure 3: Comparison of the true solution with a numerical solution using Euler's and Heun's methods

0.8 Finite Difference method

$$\frac{dy}{dx} = \frac{y_{i+1} - y_i}{h}, \quad (24)$$

$$\frac{d^2y}{dx^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}. \quad (25)$$

The finite difference expression for the fourth-order derivative is given by:

$$\frac{d^4y}{dx^4} = \frac{y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}}{h^4}.$$

In the finite difference method, the derivatives in the differential equation are approximated using the finite difference formulas. We can divide the the interval of $[a,b]$ into n equal sub-intervals of length h as shown in the following figure. The problem formulation is such that all the methods are implied on the ODE's.

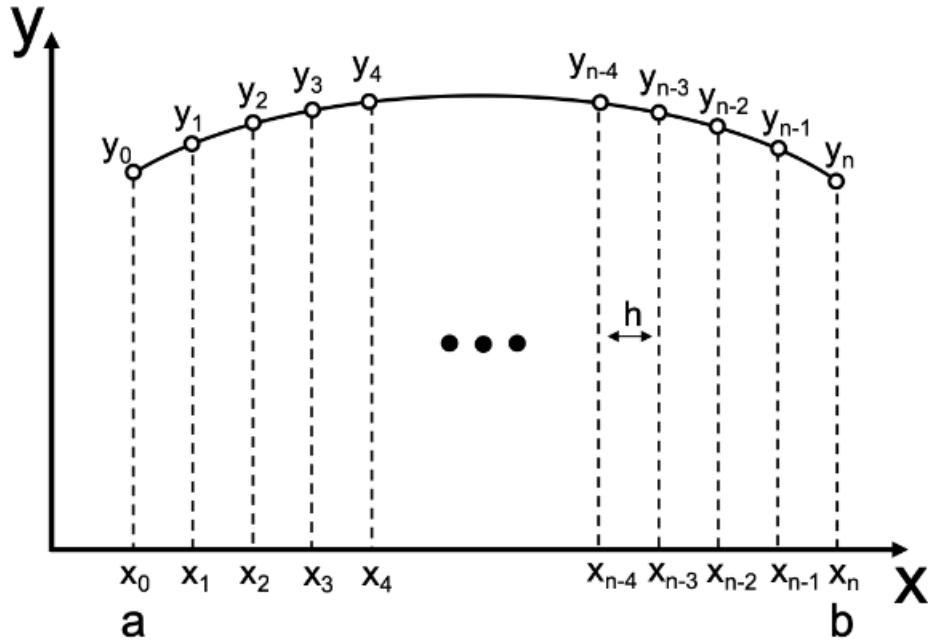


Figure 4: Finite difference method

0.9 Solving the ODE's

The problem are using the ODE's formulated and taking into account the initial values of the some unknowns as standard values:

Parameter	Value (taken)
K_s	57.24g/L
K_d	0.48 day^{-1}
μ_{max}	0.1058 day^{-1}
Y_{xis}	0.226
X_O	61.1g/L
S_O	74.51g/L
Y_p	41.11

Table 2: Required Kinetic parameters

The below equations are for calculating $d[S]/dt$

$$\frac{d[S]}{dt} = -\frac{\mu[X]}{Y_{xis}} \quad (26)$$

$$\frac{d[S]}{dt} = -\mu \left(([S_0] - [S]) + \frac{[X_0]}{Y_{xis}} \right) \quad (27)$$

$$\frac{d[S]}{dt} = -\frac{\mu_{max}[S]}{K_s + [S]} \left(([S_0] - [S]) + \frac{[X_0]}{Y_{xis}} \right) \quad (28)$$

$$\frac{d[S]}{dt} = -\frac{0.1085[S]}{57.24 + [S]} \left(74.51 - [S] + \frac{61.1}{0.226} \right) \quad (29)$$

The below equations are for calculating $d[X]/dt$

$$\frac{d[X]}{dt} = \left(\frac{\mu_{max}[S]}{K_s + [S]} - K_d \right) [X] \quad (30)$$

$$\frac{d[X]}{dt} = \left(\frac{\mu_{max}([S_0] - \frac{([X]-[X_0])}{Y_{XIS}})}{K_s + [S_0] - \frac{([X]-[X_0])}{Y_{XIS}}} - K_d \right) \cdot [X] \quad (31)$$

$$\frac{d[X]}{dt} = (0.1085 \left(\frac{74.51 - \frac{[X]-61.1}{0.226}}{57.24 + 74.51 - \frac{[X]-61.1}{0.226}} \right) - 0.48) \cdot [X] \quad (32)$$

The below equations are for calculating $d[P]/dt$

$$\frac{d[P]}{dt} = Y_p \mu [X] \quad (33)$$

$$\frac{d[P]}{dt} = Y_p \left(\frac{\mu_{max}[S]}{K_s + [S]} \right) Y_{xis} ([S_0] - [S] + [X_0]) \quad (34)$$

$$\frac{d[P]}{dt} = 41.11 \left(\frac{0.1085[S]}{57.24 + [S]} \right) (0.226(74.51 - [S]) + 61.1) \quad (35)$$

RESULT AND DISCUSSION

0.10 Data interpreted

From the numerical methods and analysis we got following results and values through Euler and Runge Kutta method.

Time	[X]	[S]
0	1.8	34
1	1.87469	22.6164
2	1.95245	14.0634
3	2.03341	8.20669
4	2.11769	4.55922
5	2.20544	2.45423
6	2.29678	1.29879
7	2.39187	0.681687
8	2.49085	0.356428
9	2.59388	0.186029
10	2.70112	0.0970091
11	2.81274	0.0505659
12	2.92891	0.0263517
13	3.04981	0.0137313
14	3.17563	0.00715462
15	3.30656	0.00372778
16	3.44281	0.00194226
17	3.584583	0.00101195
18	3.88557	0.000527244
19	4.04523	0.000274702
20	4.21133	0.000143124
21	4.38411	7.45695e-005
22	4.56383	3.88517e-005
23	4.75075	2.02423e-005
24	4.94515	1.05465e-005
25	5.14731	5.49489e-006
26	5.35753	2.86291e-006
27	5.57609	1.49162e-006
28	5.80333	7.77153e-007
29	6.03956	4.04908e-007
30	6.2851	2.10962e-007
31	6.54031	1.09914e-007

Time	[X]	[S]
32	6.80553	5.72669e-008
33	7.08113	2.98369e-008
34	7.36747	1.55454e-008
35	7.66494	8.09938e-009
36	7.97393	4.21989e-009
37	8.29484	2.19862e-009
38	8.62809	1.14551e-009
39	8.9741	5.96828e-010
40	9.33329	3.10956e-010
41	9.70611	1.62012e-010
42	10.093	8.44106e-011
43	10.4944	4.39791e-011
44	10.9108	2.29137e-011
45	11.3427	1.19384e-011
46	11.7906	6.22006e-012
47	12.2548	3.24074e-012
48	12.736	1.68847e-012
49	13.2346	8.79715e-013
50	13.7511	4.58344e-013
51	14.2859	2.38804e-013
52	14.8396	1.2442e-013
53	15.4127	6.48245e-014
54	16.0056	3.37745e-014
55	16.6188	1.7597e-014
56	17.2528	9.16827e-015
57	17.9079	4.7768e-015
58	18.5847	2.48878e-015
59	19.2835	1.29669e-015
60	20.0047	6.75592e-016
61	20.7486	3.51993e-016
62	21.5155	1.83393e-016
63	22.3057	9.55504e-017
64	23.1193	4.97831e-017
65	23.9566	2.59377e-017
66	24.8177	1.35139e-017
67	25.7024	7.04093e-018
68	26.6108	3.66842e-018
69	27.5427	1.9113e-018
70	28.4978	9.95813e-019
71	29.4758	5.18832e-019
72	30.4761	2.70319e-019
73	31.498	1.4084e-019
74	32.5409	7.33796e-020
75	33.6037	3.82318e-020
76	34.6852	1.99193e-020
77	35.7843	1.03782e-020
78	36.8992	5.4072e-021

Time	[X]	[S]
79	38.0283	2.81723e-021
80	39.1696	1.46781e-021
81	40.3209	7.64751e-022
82	41.4796	3.98446e-022
83	42.6432	2.07596e-022
84	43.8086	1.0816e-022
85	44.9726	5.63531e-023
86	46.1319	2.93607e-023
87	47.2826	1.52973e-023
88	48.4212	7.97013e-024
89	49.5434	4.15255e-024
90	50.6454	2.16354e-024
91	51.723	1.12723e-024
92	52.772	5.87304e-025
93	53.7886	3.05993e-025
94	54.769	1.59427e-025
95	55.7096	8.30636e-026
96	56.6072	4.32773e-026
97	57.4592	2.25481e-026
98	58.2633	1.17479e-026
99	59.018	6.1208e-027
100	59.7222	3.18902e-027
101	60.3756	1.66152e-027
102	60.9783	8.65677e-028
103	61.5313	4.5103e-028
104	62.0358	2.34993e-028
105	62.4937	1.22434e-028
106	62.9073	6.37901e-029
107	63.2791	3.32355e-029
108	63.6118	1.73162e-029
109	63.9083	9.02196e-030
110	64.1716	4.70057e-030
111	64.4046	2.44906e-030
112	64.61	1.276e-030
113	64.7907	6.64811e-031
114	64.9493	3.46376e-031
115	65.088	1.80467e-031
116	65.2092	9.40256e-032
117	65.3148	4.89887e-032
118	65.4068	2.55238e-032
119	65.4867	1.32982e-032
120	65.5561	6.92857e-033
121	65.6163	3.60988e-033
122	65.6684	1.8808e-033
123	65.7135	9.79922e-034
124	65.7525	5.10553e-034
125	65.7863	2.66005e-034

Time	[X]	[S]
126	65.8154	1.38592e-034
127	65.8406	7.22086e-035
128	65.8623	3.76217e-035
129	65.8811	1.96014e-035
130	65.8973	1.02126e-035
131	65.9112	5.32091e-036
132	65.9233	2.77227e-036
133	65.9336	1.44439e-036
134	65.9426	7.52547e-037
135	65.9503	3.92088e-037
136	65.9569	2.04283e-037
137	65.9626	1.06434e-037
138	65.9675	5.54538e-038
139	65.9718	2.88922e-038
140	65.9754	1.50532e-038
141	65.9786	7.84294e-039
142	65.9813	4.08628e-039
143	65.9836	2.12901e-039
144	65.9857	1.10924e-039
145	65.9874	5.77931e-040
146	65.9889	3.0111e-040
147	65.9902	1.56883e-040
148	65.9913	8.17381e-041
149	65.9922	4.25867e-041
150	65.993	2.21882e-041

Table 3: Interpreted data of [X] and [S]

0.11 Graphical analysis

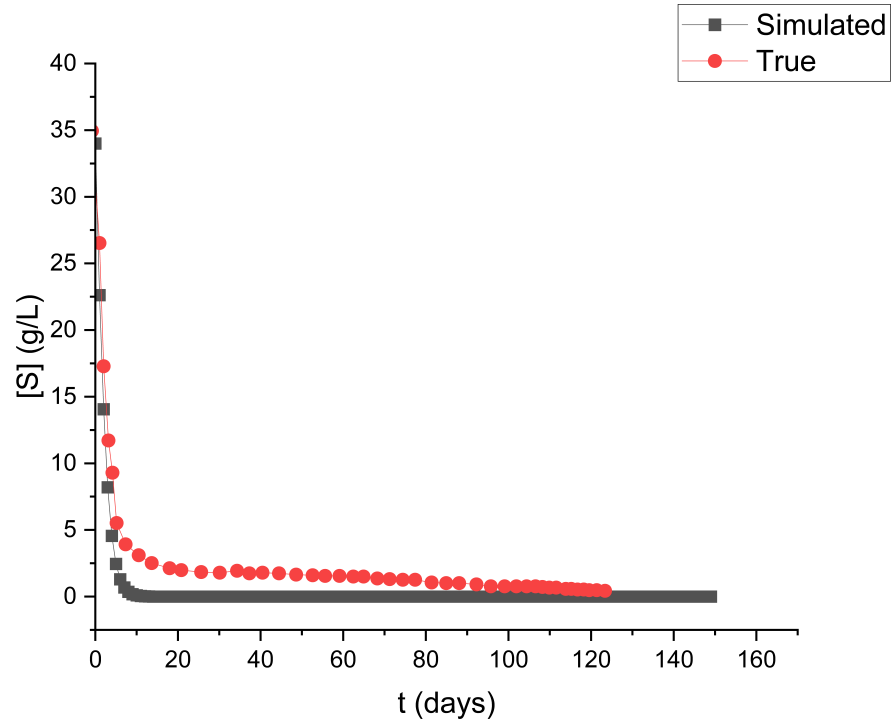


Figure 5: $[S]$ (concentration of substrate) vs time

The simulations employ the Monod model. For simple substrates, the accuracy of the Monod model is good. The variations in substrate concentration throughout time are depicted in Figure (5). The input substrate breaks down over time, releasing the energy required to create, develop, and allow microbes to survive. Figure (5) shows that the substrate concentration drops to zero in roughly seventy days.

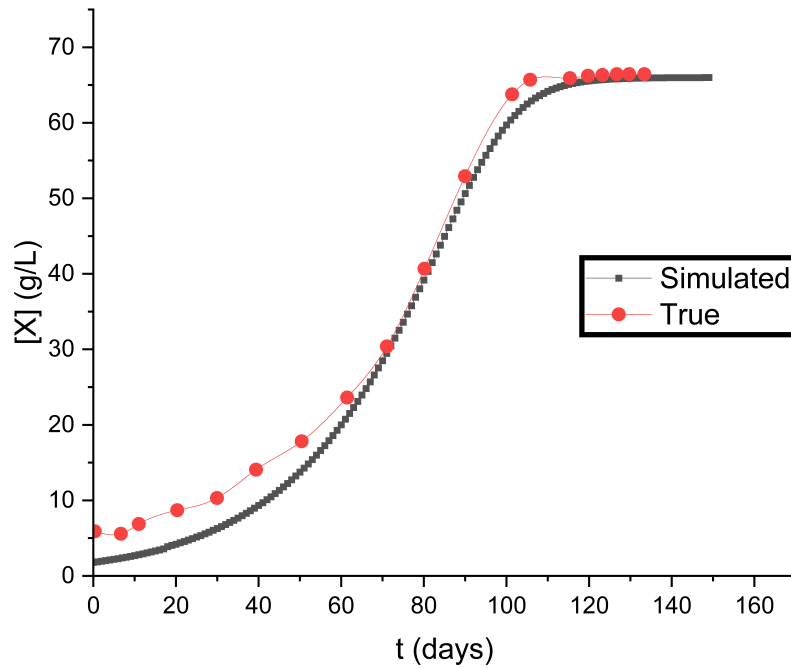


Figure 6: $[X]$ (concentration of microorganisms) vs time

The variations in the microorganism concentration over time are depicted in Figure (6). Microorganisms begin to grow and proliferate as the time and amount of input substrate consumed increases, until they attain a constant value with decreasing substrate concentration reach the dying stage once it has been fully consumed. Figures (5) and (6) demonstrate that concurrently as the substrate is completely consumed over time, the microbial production also comes to a stop and stays at that value. Figure 3 depicts the variations in the concentration of methane gas accumulated over time. Figure 3 illustrates how methane gas output increases over time rises until the production of the substrate reaches a particular level and stays there.

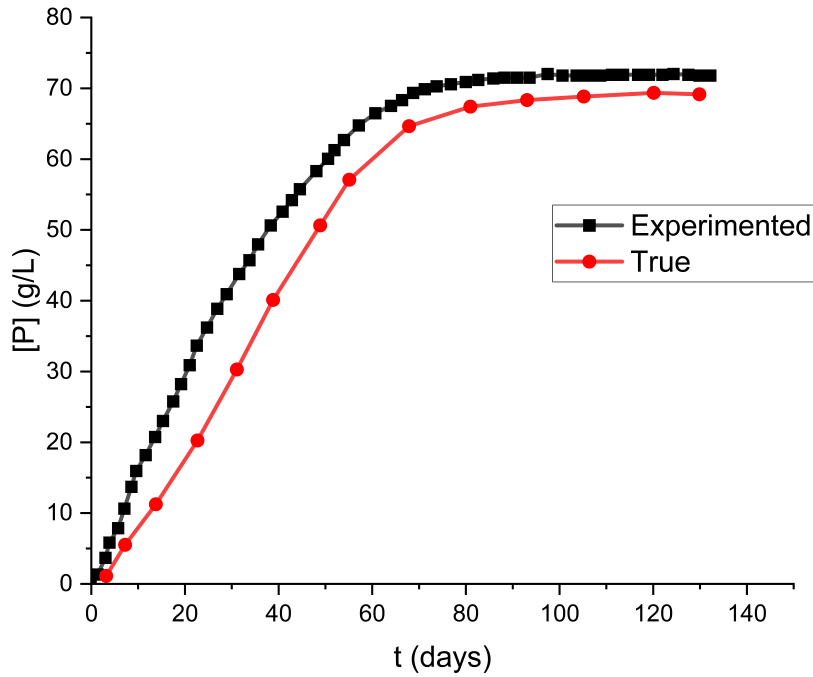


Figure 7: [X] (concentration of methane gas) vs time

The modelling can be used to accurately predict the amount of methane production from waste and animal waste, as demonstrated by the comparison of the obtained results with the reference laboratory results [20–22]. The suggested mean relative error 53.8 percent is the model for estimating the daily production of methane and laboratory results. Given that the suggested model and laboratory data agree well, the concentration of the effect of primary microorganisms on the reactor’s methane production rate is examined. Figure (8) illustrates how the starting concentration of microorganisms affects the substrate’s sustained focus over time. Figure demonstrates that an increase in the initial concentration of the rate at which substrate is consumed increases due to microorganisms. This permits the initial entry of the biogas-producing bacteria into the fermentation medium and earlier starts the decomposition process. That is to say, raising the initial rate at which the input substrate breaks down is accelerated by the concentration of microorganisms to be employed, or if their initial concentration falls, it takes time for the microbes to continue growing until the substrate breaks down and biogas is generated. This instance is known as the stage of delay [18, 23]. Figure (8) indicates that the initial focus of microorganisms declines while the latency phase lengthens.

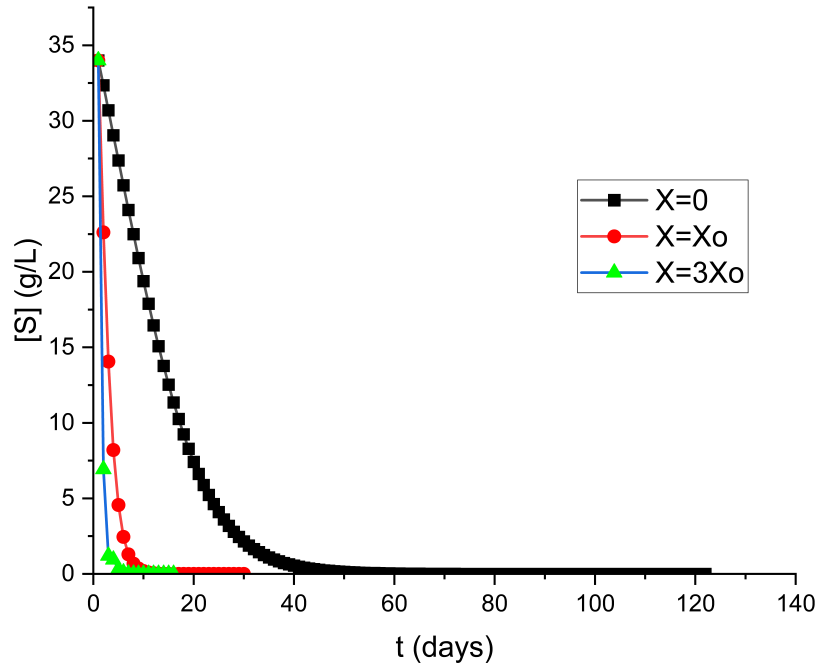


Figure 8: Variation of inlet substrate concentration with time at a different load of microorganisms.

Based on Figure (9), the growth rate of microorganisms increases with an increase in initial concentration, leading to a faster breakdown of the input substrate. Consequently, it makes sense to assume that by raising the starting microorganism concentration, it can take less time to complete the final product's production. As Figure (9) illustrates, the development consisting of four stages for microorganisms. Using the concentration $X = X_o = 0$ as an illustration, There are two stages in microorganism growth: **the exponential stage, the growth rate phase** comes right after the exponential phase. The stage of stagnation where the growth of microorganisms is demonstrated As the starting microorganism concentration falls, Figure (9) illustrates that the growth rate slows down and the stagnation phase is postponed as the latency phase lengthens. Conversely, increasing the microorganism concentration has the exact opposite effect.

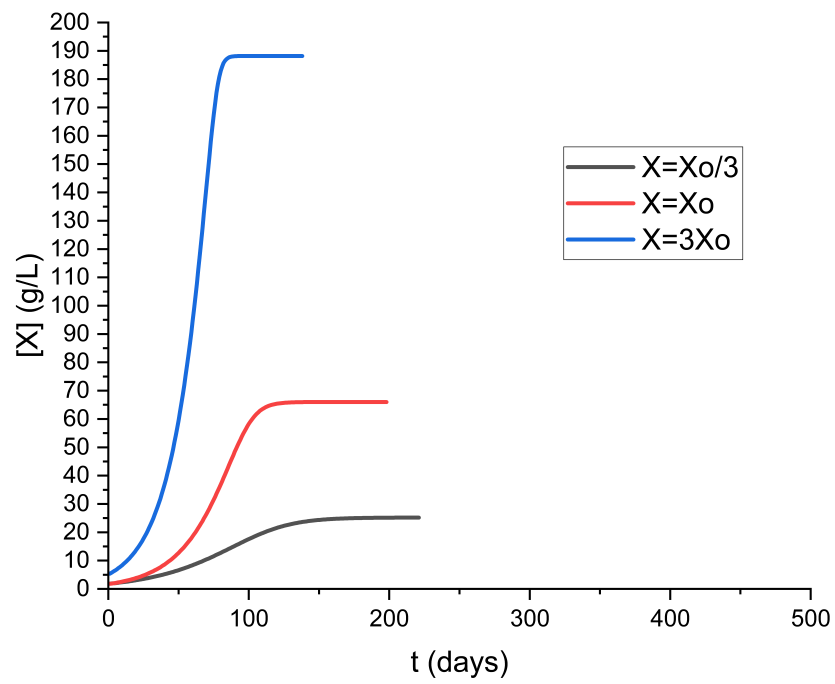


Figure 9: The effect of a different load of microorganisms on microorganisms' concentration

CONCLUSION

0.12 Path forward

So in the above research we tried to focus on the advancement of this process more efficiently and accurately, we tried to implement more accurate methods like **Runge-Kutta method of fourth order, Euler method, Huen's method, Finite difference method** not like the previous classical methods to estimate the bacterial growth and substrate degradation and generation of the methane gas from the bioreactors. We also used the interpolation to generate the curve through the points interpreted through our respective codes. We tried to maintain the uniqueness of the term paper and only explored bunch of research papers to expand our knowledge in this field. We tried to include as much as numerical methods from our course so that we get to learn from our topics in real life application based problems. Our research proved to be out some helpful in getting accurate curves in comparison with the reference graph in research papers and the error come to be out drastically less. One of the difficulties we faced in our run was that we could find the method to find the solution to ODE using more than 2 variables so we took it from the another paper as reference and moved with that but we tried it analytically and got the result close to that. This was a great experience from our side and also we got to know many things from this term paper. Our term paper will provide more newer approach and method to solve the ODE's more efficiently to get minimum error.

0.13 Conclusion

Studies on biogas production theory have been carried out as a result of the necessity to use renewable energy. It has been thought about producing biogas from fats, wastewater, plant wastes, etc. Owing to the fact that this study's livestock manure produced biogas, a constant animal waste has been used to model and simulate a biogas production reactor. The process of digestion in addition to employing a basic kinetic model for biogas production simulation, We have solved the equations using the traditional, elementary Runge Kutta method of order four. nonetheless, the model's output is consistent with the biogas production process's laboratory results. Using animal faeces. The impact of the starting microorganism concentration on the following research, the following conclusions about methane production were made. The formula for the first degree of microorganism production speed is observed. After the substrate is finished consumption, the growth of microorganisms likewise comes to an end and stabilises. As the Methane gas production and the substrate's rate of decomposition are dependent on length of stay. When the starting concentration of microorganisms is increased, the end product's less time should be needed for production. Additionally, the delay phase is shorter, and as a result, there is a reduction in the time needed for the input substrate to completely decompose. Modification of the first the concentration of microorganisms has no effect on the production of the finished product and only speeds up the fermentation of the substrate, boosts the output of biogas, and decreases the duration

needed to focus to its highest level. Put differently, the main microorganisms act as energizers. Consequently, the reaction that produces biogas from animal waste is vehicle-catalytic.

0.14 Self-assessment

Our group has used a wide range of complex numerical techniques to conduct a thorough investigation of the bacterial growth processes inside a discontinuous bioreactor for this term paper. What makes our method work is that we solve the complex differential equations that come with modelling bacterial growth by skill-fully applying well-known methods like Runge-Kutta, Heun's method, finite difference, and Euler's method. The precision, accuracy, and computational efficiency of our approach set it apart. We demonstrate the robustness of our simulations and the subtle variations in their outputs by carefully contrasting the results from these various numerical techniques. We also discuss the inclusion of sensitivity analyses to clarify the effects of parameter variations and dive into stability analyses to validate the robustness of our selected methods. The bacterial growth patterns are vividly represented graphically, which serves as a visual proof of the effectiveness of our numerical strategies. Through the discussion of obstacles faced, admission of constraints, and recommendations for further investigation, our group highlights the careful attention to detail and creativity required to solve difficult differential equations with ease, which advances our knowledge of the dynamics of bacterial growth in a discontinuous bioreactor. So, we are apply for Level-2 as our codes are running and giving desired result close to the true solution. This promotes us to **fulfill the requirements of the Level-1 and move to the Level-2**, we believe that our research has got some of the accuracy and matches to the given references. **This will give how the microorganisms grow inside the bioreactor and produce the biomass and study that kinetics in a newer way**, we studied many research paper relating our topic and we got new ideas to implement in our paper.

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